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Van Erden (F. W.) Flora Batava. Aflev. 307, 308. 4to. *Leiden* [1894]. The Netherlands Government.

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February 7, 1895.

Sir JOHN EVANS, K.C.B., D.C.L., LL.D., Vice-President and Treasurer, in the Chair.

A List of the Presents received was laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "The Oscillations of a Rotating Ellipsoidal Shell containing Fluid." By S. S. HOUGH. Communicated by Sir R. S. BALL, F.R.S. Received January 18, 1895.

(Abstract.)

The paper contains an application of the analysis used by M. Poincaré, in his memoir "Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation,"* to the determination of the free oscillations of a system consisting of a fluid mass contained within a rigid ellipsoidal envelope, rotating about one of its principal axes. It is found that, when such a system is oscillating in one of its fundamental modes, the disturbances of the fluid are all expressible by means of Lamé functions, the functions involved being all of the same order; and a method of obtaining the frequencies of these oscillations, similar to that used by M. Poincaré for a fluid ellipsoid with a free surface, is given.

The oscillations, however, which involve Lamé functions of the second order, demand exceptional treatment in consequence of the fact that these alone imply any disturbance of the containing shell. Poincaré's analysis, with slight modifications to adapt it to the

* 'Acta Mathematica,' vol. 7.

problem in hand, enables us to determine an expression for the fluid pressure at all points on the boundary in terms of the disturbances communicated to the shell. From this, the couples on the shell, due to the fluid pressure, are estimated and introduced into the dynamical equations of motion of the shell. A frequency equation of the 6th degree is derived, apparently involving three fundamental modes of oscillation. The equation, however, is found to be satisfied by the frequency of rotation of the system, and the corresponding oscillation is shown not to be real but to arise (analytically) only in consequence of the motion of the axes of reference. We are left with two fundamental modes.

The case where the inertia of the shell is negligible, compared with that of the fluid, is of analytical interest, and can be approximately realised physically by means of a liquid gyrostat ('Nature,' vol. 15, p. 297) mounted in such a way that its centre of gravity is held at rest. When the axis of rotation is an axis of symmetry, the roots of the frequency equation will be real, and the motion therefore stable, either when this axis is the least axis, or when it exceeds three times the equatorial radius. When, however, the figure is not one of revolution, the analytical conditions of stability are not so simply expressible, but they will always be satisfied when the axis of rotation is the least axis, or when it exceeds three times either of the other axes.

On taking into account the inertia of the shell, the discussion is confined to the case where the ellipsoid is approximately spherical, and the solutions of the frequency equation then assume a simple form. Of the two modes of oscillations, the motion of the shell in one is analogous to the motion of a rigid body when slightly disturbed from a motion of rotation about a principal axis, but the period is found to be shorter than it would be were the fluid solidified; the other exists only in consequence of the contained fluid.

The former of these presents the greater interest. It has been supposed that if the axis of rotation of the earth were displaced from its axis of figure, an oscillatory motion would ensue which would give rise to a variation in the latitude of places on the earth's surface in a period of 305 days. Recent observations (*vide* Chandler, 'Astronomical Journal,' vols. 11, 12) have proved that such an oscillation is taking place, but that the theoretical estimate of the period is considerably too short. This paper was undertaken with the object of investigating whether the extension of the period could be explained by supposing that the earth possessed a fluid interior, in accordance with a suggestion made by M. Folie ('Acta Mathematica,' vol. 16). It is shown that the hypothesis of a fluid interior leads to a result directly opposite to that which observation requires, and that, therefore, the discovery of the variations of latitude so far from

establishing the existence of a fluid interior, as supposed by M. Folie, rather affords an additional reason for discarding this hypothesis.

II. "On the Abelian System of Differential Equations, and their Rational and Integral Algebraic Integrals, with a Discussion of the Periodicity of Abelian Functions." By Rev. W. R. Westropp Roberts. Communicated by Rev. G. Salmon, D.D., F.R.S. Received January 17, 1895.

(Abstract.)

Before entering on the discussion of the Abelian system of differential equations, I treat of some general algebraic theorems having reference to the differences of various sets of "facients," and give a wider definition to the term "source," hitherto used to signify the source of a covariant, and treat of two operators, δ and Δ.

I then show how, by forming what I call a "square-matrix," all the conditions can be obtained which are fulfilled when a polynomial $f(2)$ of the degree $2n$ in z is a perfect square. With regard to these conditions, I remark that any one of them being given all the others can be found by successive operations of the operator δ.

I next treat of the system of differential equations termed "Abelian," in which there are m quantities and $m-1$ equations, comprehended in the typical form

$$\Sigma \frac{z^i dz}{\sqrt{f(z)}} = 0,$$

where Σ relates to the m quantities $z_1, z_2, \ldots, z_m$, and i may have any integer value from $i = 0$ to $i = m-2$, it being understood that $f(z)$ is a polynomial of the degree $2m$ in z; and I show that, if $f(z) \equiv z^{2m} + P_1 z^{2m-1} + P_2 z^{2m-2} + \ldots P_{2m}$, be reduced to the degree $2m-2$ in z in the following manner—

$$f(z) + \{\phi(z)\}^2 - 2\phi(z) \,.\, L(z) \equiv F(z),$$

where

$$\phi(z) \equiv (z-z_1)(z-z_2)\ldots(z-z_m) \equiv z^m + p_1 z^{m-1} + \ldots p_m,$$

and
$$L(z) = z^m + \frac{P_1}{2} z^{m-1} + \lambda_2 z^{m-2} + \lambda_3 z^{m-3} + \ldots + \lambda_m,$$

$\lambda_2, \lambda_3, \ldots, \lambda_m$ being $m-1$ arbitrary constants, all the rational and integral algebraic integrals of the Abelian system

$$\Sigma \frac{z^i dz}{\sqrt{f(z)}} = 0$$